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Research on the mechanism of curvature about complex surface based on five-axis flank milling

Zhiyong Song[a,b]*, Liping Wang[a], Wenjian Tao[b], Qingzhao Li[b], Wei Wang[a]

[a] *School of Mechatronics Engineering, University of Electronic Science and Technology of China, No. 2006, Xiyuan Ave, West Hi-Tech Zone, ChengDu 611731, Sichuan, China*
[b] *AVIC ChengDu Aircraft Industry (Group) Co.,Ltd, ChengDu 610091, Sichuan, China*

* Corresponding author. Tel.: + 028-8740-7433; fax: + 028-8740-6536. *E-mail address:* 478614690@qq.com

Abstract

Curvature and curvature continuity, as two of the main geometry characteristics of complex surface, cause significant influence to the surface quality in the five-axis flank milling. Firstly, the curvature continuity grade in different complex surfaces have been defined accurately by introducing the basic concept about curvature and curvature continuity. Secondly, the relationship between curvature, curvature continuity and the machine tools movement are analyzed theoretically, it can be gotten that different curvature and continuity may lead to different machine tools movement law. Lastly, simulation results verify the correction about theoretical analysis, which is coincident with the theoretical analysis. The mechanism between curvature, curvature continuity and machine tools movement, contributes to achieve the optimization and improvement of complex surface quality.

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Keywords: curvature; curvature continuity; mechanism; five-axis flank milling

1. Introduction

With the rapid development of manufacture industry, traditional three-axis machining has been unable to meet the requirement of many products processing, five-axis CNC machine tools, as its high cutting accuracy and machining efficiency, is widely used in different kinds of products processing, especially in turbine machinery, aviation parts and complex surface, which has shown obvious advantages[1]. During the five-axis machining, the complexity of surface may lead to a few shortcomings, such as the machining error is hard to control and tool path smoothing degree is unable to meet demands. Flank milling, as a kind of important complex surface processing methods, own very excellent capabilities, especially in aerospace parts, turbine blades and ruled surface processing[2,3]. However, it may cause that the cutting quality can not meet the accuracy requirement because of a variety of geometric characteristics of complex surface. Research shows that the curvature of complex surface is a key factor for machining error, studying the mechanism of curvature, which is contribute to reveal the relationship between curvature and five-axis CNC machine tools motion and achieve the optimization and improvement of complex surface quality.

Zhenyuan JIA, et al[4] studied the relationship between the surface curvature, five-axis CNC machine tools cutting force and surface error, paper [5] proposed a flexible acceleration and deceleration NURBS interpolator based on surface curvature properties, paper [6] proposed that the tool path planning should take full account of the machine's kinematics and dynamics characteristics. However, there is no report about the relationship between the curvature and the motion of the five-axis CNC machine tools.

It is unavoidable to cut the complex surface with a variety of curvature, which cause the decline of surface quality in actual cutting, therefore we may conclude that curvature characteristics of complex surface is a weak point of the

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machine tool cutting. In this paper, we will study the motion law of machine tool based on the curvature and the curvature continuity to explain the mechanism of curvature.

2. The concept of curvature and its mechanism

2.1. The concept about curvature and curvature continuity

(1) Curve and surface curvature

Curvature is one of the main indexes to evaluate the geometric characteristics of the curve, it reflects the camber of the curve. General parameter space curve Γ equations is $r=r(t)=r(x(t), y(t), z(t))$, its curvature can be expressed as:

$$k=\frac{|r'(t)\times r''(t)|}{|r'(t)|^3}=\frac{\left[\begin{vmatrix} y'(t) & z'(t) \\ y''(t) & z''(t)\end{vmatrix}^2+\begin{vmatrix} z'(t) & x'(t) \\ z''(t) & x''(t)\end{vmatrix}^2+\begin{vmatrix} x'(t) & y'(t) \\ x''(t) & y''(t)\end{vmatrix}^2\right]^{1/2}}{\left((x'(t))^2+(y'(t))^2+(z'(t))^2\right)^{3/2}} \tag{1}$$

In the formula, $r'(t)$ and $r''(t)$ represent first derivative and second derivative of the curve Γ respectively.

The bendability of curved surface can be described by the Gauss curvature which is the product of two principal curvature of arbitrary point on the surface. The expression of space curved surface $\sum$ is $r=r(u(t),v(t))$, so the Gauss curvature of arbitrary point on the surface can be expressed as:

$$k=k_1\cdot k_2=\frac{LN-M^2}{EG-F^2} \tag{2}$$

In this formula, k_1, k_2 represent the maximum and minimum values on surface curvature in any point on all the main direction respectively, E, F, G, L, N, M represent the first and second categories of basic quantity on the surface.

(2) Curve and surface curvature continuity

The continuity of curve and surface is an evaluation way, which describes the smoothness level of two curves or two surfaces when they are connecting. According to the quality of the level of connection smoothness, the continuity can be divided into G0 continuity, G1 continuity, G2 continuity, ⋯⋯, meanwhile, the upward trend along the curve or surface indicates that the connection is more smooth. Describing and judging the curve (surface) continuous grade accurately is the basis for studying machine tools mechanism under different curvature continuity.

① G0 continuity

G0 continuity, also called position continuity, means that the contact endpoints (boundary) between a curve (surface) and another curve (surface) is coincident, and then it is considered the curve (surface) obey G0 continuity in this connection endpoint (boundary). It can be expressed as:

$$\lim_{x\to x_0^-} f_1(x)=\lim_{x\to x_0^+} f_2(x) \tag{3}$$

In this formula, the $f_1(x)$ and $f_2(x)$ indicate different curves (surfaces) respectively and the x_0 represents the endpoint (boundary). G0 continuity can be represented by the diagram shown in Figure.1.

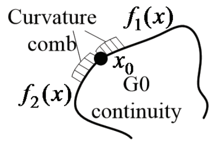


Fig.1. G0 continuity

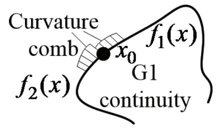


Fig.2. G1 continuity

② G1 continuity

G1 continuity is generally called tangent continuity, two curves (surfaces) share the same tangent at the connection point in the basis of obeying G0 continuity, which is more smooth than G0 continuity at the endpoint (boundary), as shown in Fig.2. The mathematical expression is:

$$\lim_{x\to x_0^-} f_1'(x)=\lim_{x\to x_0^+} f_2'(x) \tag{4}$$

$f_1'(x)$ and $f_2'(x)$ represent the tangent vector of two curves (surfaces) at the connection point and the x_0 represents the connection endpoint (boundary).

③ G2 continuity

G2 continuity is generally called curvature continuity, it ensures the same curvature for two curves (surfaces) at connection point, which is based on G0 and G1 continuity, it is more smooth than G1 continuity, as shown in Fig.3. The mathematical expression is:

$$\lim_{x\to x_0^-} f_1''(x)=\lim_{x\to x_0^+} f_2''(x) \tag{5}$$

In the formula, $f_1''(x)$ and $f_2''(x)$ represent the curvature of two curves (surfaces) at the connection point and the x_0 represents the endpoint (boundary).

From the above analysis, we may get that with the curvature continuity improving, the smooth of curve (surface) become better.

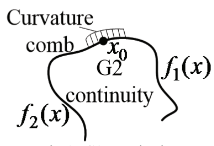


Fig.3. G2 continuity

2.2. Study on mechanism of curvature

(1) Study the relationship between curvature and acceleration

In the actual cutting, the surface quality becomes different with various curvature. The motion of cutter shaft is analyzed when it is flank milling different surfaces with various curvature, as shown in Fig.4. In the picture, tool moves along with the arc whose radius is R from position B_1 to position B_2 at the speed of V, $\Delta\theta$ is the variation angle, V_{x1}, V_{y1} and V_{x2}, V_{y2} mean the velocity in x, y directions respectively from position B_1 to position B_2.

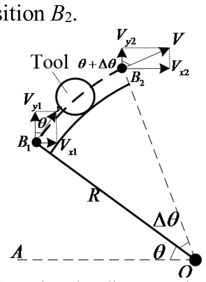


Fig. 4. Tool motion diagrammatic sketch

From Fig4, the time tool spend in total process can be calculated, as shown in the formula (6):

$$t = \frac{\Delta\theta \cdot \pi R}{V} \tag{6}$$

The velocity of tool at the point B_1, B_2 can be expressed:

$$\begin{cases} V_{x1} = V\sin\theta \\ V_{y1} = V\cos\theta \end{cases} \quad (7) \qquad \begin{cases} V_{x2} = V\sin(\theta+\Delta\theta) \\ V_{y2} = V\cos(\theta+\Delta\theta) \end{cases} \quad (8)$$

Thus, the average acceleration in x, y directions about tool can be calculated as:

$$\begin{cases} \overline{A}_x = \dfrac{V_{x2}-V_{x1}}{t} = \dfrac{V\left[\sin(\theta+\Delta\theta)-\sin(\theta)\right]\cdot 180V}{\Delta\theta\cdot\pi R} = \dfrac{180V^2k}{\pi}\cdot\dfrac{\left[\sin(\theta+\Delta\theta)-\sin(\theta)\right]}{\Delta\theta} \\ \overline{A}_y = \dfrac{V_{y2}-V_{y1}}{t} = \dfrac{V\left[\cos(\theta+\Delta\theta)-\cos(\theta)\right]\cdot 180V}{\Delta\theta\cdot\pi R} = \dfrac{180V^2k}{\pi}\cdot\dfrac{\left[\cos(\theta+\Delta\theta)-\cos(\theta)\right]}{\Delta\theta} \end{cases} \tag{9}$$

In the formula, $k = \frac{1}{R}$, when $\Delta\theta \to 0$, $\Delta t \to 0$, the average acceleration can be expressed by instantaneous acceleration:

$$\lim_{\Delta t\to 0}\overline{A_x} = \lim_{\Delta t\to 0}\frac{\Delta V_x}{\Delta t} = \frac{dv}{dt} = A_x \tag{10}$$

Therefore, calculating the limit to formula (9), when $\Delta\theta \to 0$, we can get:

$$\begin{cases} \lim\limits_{\Delta\theta\to 0}\overline{A_x} = \lim\limits_{\Delta\theta\to 0}\dfrac{180V^2k}{\pi}\cdot\dfrac{[\sin(\theta+\Delta\theta)-\sin\theta]}{\Delta\theta} = \dfrac{180V^2k}{\pi}\cdot\lim\limits_{\Delta\theta\to 0}\dfrac{[\sin(\theta+\Delta\theta)-\sin\theta]}{\Delta\theta} = \dfrac{180V^2k}{\pi}\cos\theta \\ \lim\limits_{\Delta\theta\to 0}\overline{A_y} = \lim\limits_{\Delta\theta\to 0}\dfrac{180V^2k}{\pi}\cdot\dfrac{[\cos(\theta+\Delta\theta)-\cos\theta]}{\Delta\theta} = \dfrac{180V^2k}{\pi}\cdot\lim\limits_{\Delta\theta\to 0}\dfrac{[\cos(\theta+\Delta\theta)-\cos\theta]}{\Delta\theta} = -\dfrac{180V^2k}{\pi}\sin\theta \end{cases} \tag{11}$$

It can be obtained from formula (11):

$$\begin{cases} A_x = \dfrac{180V^2k}{\pi}\cdot\cos\theta \\ A_y = -\dfrac{180V^2k}{\pi}\cdot\sin\theta \end{cases} \tag{12}$$

From equation (12), we may get the conclusion: when the feed speed is constant, acceleration increases with the augment of curvature; when the curvature is constant, acceleration augments with the feed rate increasing.

(2) Study on the relationship between curvature continuity and machine tool movement

Equation (12) reveals the relationship between curvature, feed speed and acceleration, however, the continuity level of curved surface is determined by the surface curvature, when the machine tool cutting different curvature continuity surface, its movement will also appear different law.

① The relationship between G0 continuity and machine tool movement

Fig.5 is a motion schematic when tool moves on the G0 continuous surface. In the figure, two curves OO_1, O_1O_2 obey G0 continuity at the point O_1, the tool moves from point A to point B along the curve at the feed speed of v. According to the motion relationship, the velocity, acceleration and jerk can be obtained at point A and point B, it can be expressed in formula (13) and (14) respectively.

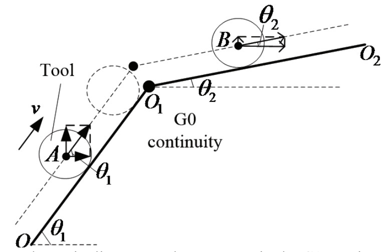


Fig. 5. The schematic diagram tool movement in the G0 continuous surface

The speed, acceleration and jerk of tool at the point A in the x, y direction is:

$$\begin{cases} V_{Ax} = v\cos\theta_1 \\ V_{Ay} = v\sin\theta_1 \end{cases} \Rightarrow \begin{cases} A_{Ax} = -v\sin\theta_1 \\ A_{Ay} = v\cos\theta_1 \end{cases} \Rightarrow \begin{cases} J_{Ax} = -v\cos\theta_1 \\ J_{Ay} = -v\sin\theta_1 \end{cases} \tag{13}$$

The speed, acceleration and jerk of tool at the point B in the x, y direction is:

$$\begin{cases} V_{Bx} = v\cos\theta_2 \\ V_{By} = v\sin\theta_2 \end{cases} \Rightarrow \begin{cases} A_{Bx} = -v\sin\theta_2 \\ A_{By} = v\cos\theta_2 \end{cases} \Rightarrow \begin{cases} J_{Bx} = -v\cos\theta_2 \\ J_{By} = -v\sin\theta_2 \end{cases} \tag{14}$$

As $\theta_1 > \theta_2$, so

$$\begin{cases} V_{Ax} < V_{Bx} \\ V_{Ay} > V_{Ay} \end{cases}, \begin{cases} A_{Ax} < A_{Bx} \\ A_{Ay} > A_{Ay} \end{cases}, \begin{cases} J_{Ax} < J_{Bx} \\ J_{Ay} > J_{Ay} \end{cases} \tag{15}$$

Formula (15) indicates that tool's velocity, acceleration and jerk changes at G0 continuous point, so the tool impulse can be expressed as:

$$P = mv,\ I = P_B - P_A = \begin{cases} m(v_{Bx}-v_{Ax}) = mv(\cos\theta_2 - \cos\theta_1) \\ m(v_{By}-v_{Ay}) = mv(\sin\theta_2 - \sin\theta_1) \end{cases} \tag{16}$$

From equation (16), the tool's impulse has changed due to the G0 continuity at the point O_1, the change of impulse will cause impact to machine, the inertial force can be expressed as:

$$f = \frac{m(v_2 - v_1)}{t} \tag{17}$$

Equation (17) shows that the inertial force tends to infinity in theory as the velocity mutation in a short time, which will cause great impact to the tool. It can be concluded G0 continuity result in the change of tool velocity, acceleration and jerk and cause impact to the machine.

② The relationship between G1, G2 continuity and machine tool movement

As shown in Fig.6, the tool moves from point A to B along the surface AB at the feed speed of v. At the point B, the tool experience three different surfaces BC, BD and BE respectively, the connecting segments of surface AB and BC obey G1 continuity, the connecting segments of surface AB and BD obey inflexion continuity as the surface concavity and convexity is different at the point B, which is the special case of G1 continuity. Meanwhile, the connection section AB and BE obey G2 continuity. The tool motion situation on three different surfaces can be analyzed respectively:

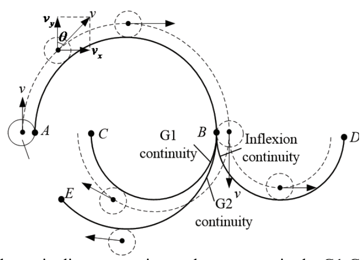
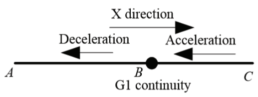
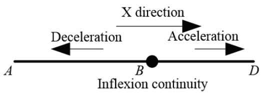


Fig.6. The schematic diagram cutting tool movement in the G1,G2 continuous surface

When the tool moving from surface *AB* to *BC* at the feed speed of *v*, the acceleration value changes at the point *B* as the change of surface curvature, but the direction dose not change as the surface concavity and convexity is same according to the part (1) in chapter 2.2. However, when the tool moving from *AB* to *BD*, the acceleration direction changes reversely, the value also changes as the surface changes from concavity to convexity. The schematic diagram about acceleration direction is shown in Fig.7.



(a) Schematic diagram about acceleration motion of G1 continuity in X direction



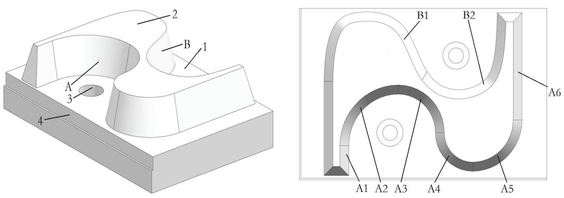
(b) Schematic diagram about acceleration motion of inflexion continuous in X direction

Fig.7. The acceleration direction of tool in the X axis direction after passing through the *B* point (G1 continuity & inflection continuous)

Compared to the G1 continuity and inflection continuity, the G2 continuity is more smooth, there is no acceleration mutation as the curvature is constant, which will not cause the impact to machine tool.

3. Simulation analysis

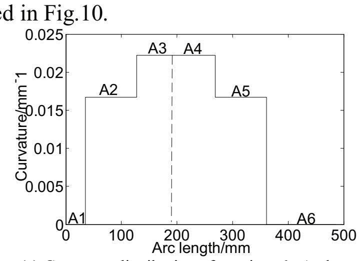
In order to verify the correctness of theoretical analysis in chapter 2.2, we construct a new specimen, as shown in Fig.8, which own all the curvature properties. Specimen consists of surface A and surface B, surface A is formed by six different surfaces with constant curvature, they are A1, A2, A3, A4, A5 and A6, the curvature of surface A1 and A6 are zeros, so the surface A is called constant curvature surface. Surface B is a ruled surface constructed by three quasi-uniform B-spline curve, whose curvature is changing constantly, surface B is divided into two surfaces: B1 and B2 according to the concavity and convexity of surface, so surface B is called various curvature surface, as shown in Fig.8. The specimen can be used to test the surface processing capacity of the machine by using the servo system simulation platform of the five-axis CNC machine tool and simulate the processing of the specimen, analyze the motion characteristics of each axis during processing, then comparing with the theoretical analysis results in chapter 2.2.



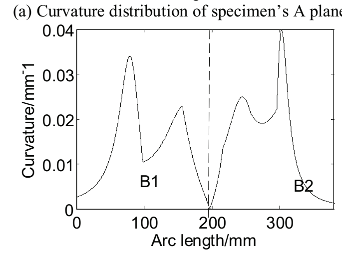
(a) Three-dimensional view of the specimen (b) A plan view of the specimen

Fig. 8. Specimen used to test surfaces processing capacity of the machine

All constant curvature surfaces constituted surface A obey G2 continuity, the junction combined each constant curvature surface obey G1 continuity, the junction between surface A3 and A4 obey inflexion continuity, the curvature distribution of specimen's A plane is showed in figure 9(a), the mean curvature is 0.01944mm^{-1}. The junction between surface B1 and B2 obey inflexion continuity and other surfaces obey G2 continuity in surface B, the curvature distribution of specimen's B plane is showed in figure 9(b), From the picture, we can see that the curvature distribution is various and the mean curvature is 0.01692mm^{-1}. The profile of specimen curvature continuity is showed in Fig.10.



(a) Curvature distribution of specimen's A plane



(b) Curvature distribution of specimen's B plane

Figure.9. Curvature distribution of specimen

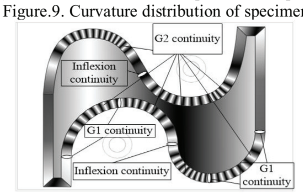
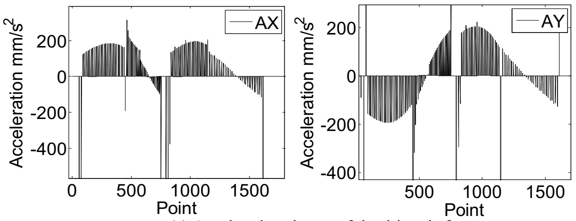


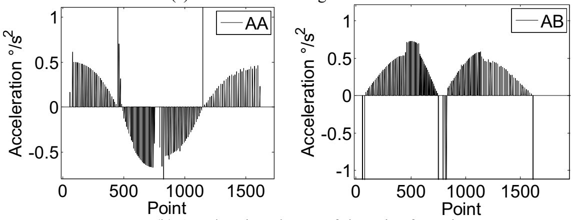
Fig.10. Profile of specimen curvature continuity

(1) Simulation about the relationship acceleration and curvature

The tools' movement at different curvatures and curvature continuity can be gotten by entering the NC code about the specimen into the machine tools servo simulation platform to simulate the CNC machine tool motion during the processing. The acceleration change diagram about drive shaft and rotation shaft in surface A and surface B are shown in Fig 11 and Fig 12 respectively.

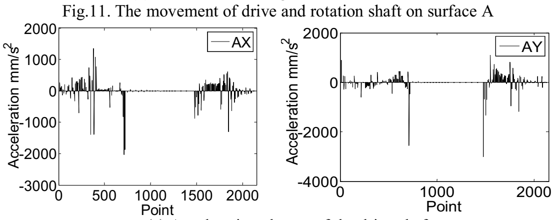


(a) Acceleration change of the drive shaft

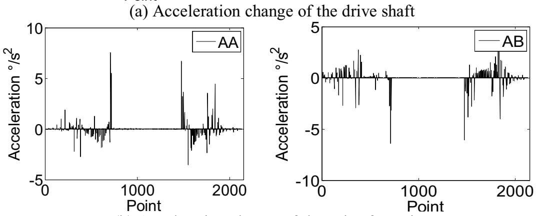


(b) Acceleration change of the axis of rotation

Fig.11. The movement of drive and rotation shaft on surface A



(a) Acceleration change of the drive shaft



(b) Acceleration change of the axis of rotation

Fig.12. The movement of drive and rotation shaft on surface B

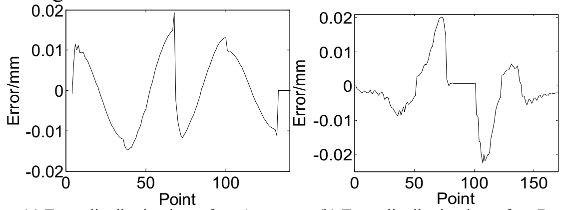
From Fig.11 and Fig.12, it can be gotten that the acceleration is large at the large curvature, the acceleration and curvature are positive correlation in the constant speed rate, which verify the conclusions in chapter 2.2.

(2) Simulation on the relationship between curvature continuity and machine kinematics

From Fig.11, as the curvature mutation, machine tool's each axis acceleration also mutate at the junction of G1 continuity, the mutation and the curvature value are positive correlation; in the inflection continuity, the acceleration direction changes reversely, the value also changes as the surface changing from concavity to convexity, which is coincident with theoretical analysis in chapter 2.2; in G2 continuity with constant curvature, there is no mutation acceleration. In Fig.12, the same results can be gotten as Fig.11.

On the simulation platform, the normal error of after-cutting specimen is measured by taking 45mm, 60mm, 75mm height line with 25 measurement points respectively in the Z direction at surface A and surface B respectively.

The measurement error is shown in Fig.13. (a) and (b) represent error distribution in the surface A and surface B respectively, from this figure, there are many mutation errors in G1 continuity, the error and mutation value are positive correlation; in the inflection continuity, error changes reversely; In the G2 continuity with constant curvature, there is no mutation error. Compared with Fig.13(a) and (b), we can get that during the processing of constant curvature and various curvature surface with similar mean curvature, the error milling various curvature surface is larger than the error milling constant curvature surface.



(a) Error distribution in surface A (b) Error distribution in surface B

Fig. 13. Error distribution of surface A and B

4. Conclusions

Through theoretical analysis and simulation about the relationship between curvature, curvature continuity and motion, the following conclusions can be gotten: (1) Acceleration and curvature value are positive correlation, when the feed speed is constant; (2) The velocity, acceleration and jerk at G0 continuity break suddenly, which may cause largest impact to machine tools; (3) At G1 continuity, the acceleration mutation and the curvature mutation are positive correlation, as well as the error mutation; (4) In inflection continuity, the acceleration direction changes reversely, which cause the error reverse; (5) There is no acceleration and error mutation at the G2 continuity with constant curvature, which is a very ideal continuity between curves (surfaces).

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